

Abstracts of the Barcelona Number Theory Seminar  
Seminari de Teoria de Nombres (UB-UAB-UPC)  
**STNB2016**, 30a ed.



# Abstracts of the STNB2016

## BARCELONA NUMBER THEORY SEMINAR

Facultat de Matemàtiques, Universitat de Barcelona  
January 25 - 29, 2016

Devoted to celebrate 30th edition of STNB  
and Prof. Pilar Bayer's 70th Birthday

Organising Committee: M. Alsina, F. Bars, A. Travesa

30 ANYS!

$\rho: \text{Gal}(\mathbb{Q}/\mathbb{Q}) \rightarrow \text{Aut}(E)$

$X^P + Y^P = Z^P$

$L(f_s) = \sum_{n \geq 1} \frac{a_n}{n^s}$

$\theta(z, z') = \sum_{n \geq 0} e^{2\pi i n z + \pi i n^2 z'}$

$\gamma(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \frac{1}{1-p^{-s}}$

$\gamma^2 = 4x^3 - g_2x - g_3$

$\lim_{s \rightarrow 1} (s-1) \zeta_k(s) = \frac{2^k (2\pi)^k h R}{\omega |\mathcal{D}_k|^k}$

STNB 2016  
SEMINARI DE TEORIA DE NOMES DE BARCELONA  
BARCELONA NUMBER THEORY SEMINAR (UB-UAB-UPC)

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## Introduction

The Seminari de Teoria de Nombres (UB-UAB-UPC) is a group of researchers in number theory mainly from Universitat de Barcelona, Universitat Autònoma de Barcelona and Universitat Politècnica de Catalunya.

Since 1986, this research group has been organising activities as courses, workshops and conferences in number theory in our country. The main annual activity is known as Seminari de Teoria de Nombres de Barcelona, joining researchers in this subject in Barcelona metropolitan area and attracting people from other places and foreign countries too. Thus, in 2016 we are celebrating its 30th edition. In this framework also a homage to Professor Pilar Bayer has been organised, in occasion of her 70th birthday, as a tribute to her contribution to the development of number theory in our country.

The Seminari de Teoria de Nombres de Barcelona, STNB2016, is going to be devoted to the topic of modular curves of infinite level, by one side, and to a more wide area, entitled *30 years of STNB*, by the other side. This booklet contains the abstracts of all the scheduled talks as provided by the respective speakers.

The topic *Corbes modulars de nivell infinit* has been coordinated by Santi Molina. It is developed through five talks, which are presented consecutively, preceded by a general introduction and followed by a common list of references.

The topic *30 years of STNB*, coordinated by Montserrat Alsina, Francesc Bars and Artur Travesa collects invited talks on different areas of number theory, most of them connected to topics developed through the history of the seminar. Moreover, communications by participants to STNB2016, coordinated by Francesc Bars, complement the overview of the research by the people related to STNB. Thus, this collection of abstracts is presented by alphabetical order of the speakers.

We hope you will find this information useful.

Barcelona, gener 2016  
M. Alsina, F. Bars, A. Travesa

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## Corbes Modulars de nivell infinit

**Coordinació:** SANTI MOLINA

El nostre objectiu és l'estudi de les reduccions dels possibles models semiestables de la corba  $X(p^n)$ . Per això, analitzarem l'anell local de qualsevol punt supersingular. La descripció d'aquests anells locals es fa més i més complicada a mesura que  $n$  va creixent, no obstant això, la completació del seu límit directe quan  $n$  tendeix a infinit té una descripció molt precisa. Aprofitarem aquesta descripció per a trobar models semiestables “a l’infinit”, i projectar-los a nivell finit. Per tal de descriure models semiestables “a l’infinit”, és necessari treballar amb geometria analítica perfectoïde, ja que, fent el límit quan  $n$  tendeix a infinit, ens sortim fora de la categoria on podem aplicar geometria rígida analítica.

### Introducció a les corbes modulars de nivell infinit

XEVI GUITART, Universitat de Barcelona

**ABSTRACT:** En aquesta primera xerrada enunciaré el resultat principal de l’article de J. Weinstein [1], en què es determinen les possibilitats per a les components irreductibles de les fibres especials dels models semiestables de (certes) corbes modulars. També donaré una visió general, sense entrar en detalls, de quina és l’estratègia que Weinstein segueix per a demostrar-lo. En xerrades posteriors s’aprofundirà en alguns dels aspectes més tècnics de la demostració, i per tal de facilitar aquesta tasca també aprofitaré per a introduir algunes de les nocions que hi jugaran un paper destacat, com ara el model de Katz–Mazur de la corba modular  $X(N)$ , les varietats rígides analítiques, els esquemes formals i els recobriments semiestables.

## Grups $p$ -divisibles, grups formals i deformacions

EDUARD SOTO, Universitat de Barcelona

**ABSTRACT:** En aquesta xerrada recordarem què són els esquemes, els esquemes en grup i els grups  $p$ -divisibles i veure'm com generalitzar aquests conceptes al món dels esquemes formals. Així, introduirem les generalitzacions corresponents: esquemes formals, grups formals i grups formals  $p$ -divisibles. També parlarem sobre estructures de nivell de Drinfeld associades a grups formals  $p$ -divisibles. Fixat un grup formal  $\mathcal{G}_0$  sobre  $W = W(\mathbb{F}_p)$  (amb o sense estructura de Drinfeld) estudiarem els seus possibles aixecaments (o deformacions) a anells locals noetherians complets amb cos residual  $\mathbb{F}_p$ . Per un primer  $p$  i un enter  $N \geq 5$  no divisible per  $p$  denotem per  $Y_n$  l'esquema regular sobre  $W$  que representa el problema de moduli  $[\Gamma_1(N) \cap \Gamma(p^n)]$ . L'objectiu final d'aquesta xerrada serà interpretar l'anell local completat  $\hat{\mathcal{O}}_{Y_n,x}$  al voltant d'un punt supersingular  $x \in Y_n$  com a solució d'un problema de deformació de grups  $p$ -divisibles amb estructura de Drinfeld.

## Espais de Lubin-Tate de nivell infinit

SANTI MOLINA, Centre de Recerca Matemàtica

**ABSTRACT:** Els anells locals  $\hat{\mathcal{O}}_{Y_n,x}$ , introduïts a l'anterior xerrada, es fan més i més complicats a mesura que  $n$  va creixent. Un dels resultats principals de J. Weinstein [1] és donar una descripció explícita de la completació del límit directe dels  $\hat{\mathcal{O}}_{Y_n,x}$  quan  $n$  tendeix a infinit. En aquesta xerrada explicarem aquest resultat. Per tal d'això, introduirem els espais vectorials formals i els morfismes determinant. Aquesta descripció explícita s'utilitzarà en la darrera xerrada per a trobar els desitjats models semiestables.

## Espais àdics i espais perfectoides

ALBERTO CÁMARA, University of Nottingham

**ABSTRACT:** Per tal d'entendre l'estructura de la corba modular a "nivell infinit" farem servir la teoria dels espais perfectoides de

Scholze. En aquesta xerrada farem una introducció abreujada als ingredients de geometria analítica  $p$ -àdica que són necessaris per a la teoria dels espais perfectoides de Scholze. En particular, introduirem els espais àdics de Huber i definirem els espais perfectoides de Scholze i l'equivalència “tilt”.

## Models estables de corbes de nivell infinit

XAVIER XARLES, Universitat Autònoma de Barcelona

**ABSTRACT:** Explicarem la relació entre models semiestables i certs recobriments per oberts analítics. Després farem servir la descripció explícita de la corba de moduli de nivell infinit per a trobar un d'aquests recobriments per les corbes modulars de nivell finit. A més la descripció és prou precisa que és possible descriure la reducció del model semiestable associat al recobriment anterior, tant de les seves components irreductibles com del seu graf dual.

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## Abstracts for the conference talks

**Coordinació:** M. ALSINA, F. BARS, A. TRAVESA

### Inverse Galois problem and uniform realizations

SAMUELE ANNI, University of Warwick

**ABSTRACT:** Let  $\overline{\mathbb{Q}}$  be an algebraic closure of  $\mathbb{Q}$ , let  $n$  be a positive integer and let  $\ell$  a prime number. Given a curve  $C$  over  $\mathbb{Q}$  of genus  $g$ , it is possible to define a Galois representation  $\rho : \text{Gal}(\overline{\mathbb{Q}}|\mathbb{Q}) \rightarrow \text{GSp}_{2g}(\mathbb{F}_\ell)$ , where  $\mathbb{F}_\ell$  is the finite field of  $\ell$  elements and  $\text{GSp}_{2g}$  is the general symplectic group in  $\text{GL}_{2g}$ , corresponding to the action of the absolute Galois group  $\text{Gal}(\overline{\mathbb{Q}}|\mathbb{Q})$  on the  $\ell$ -torsion points of its Jacobian variety  $J(C)$ . If  $\rho$  is surjective, then we realize  $\text{GSp}_{2g}(\mathbb{F}_\ell)$  as a Galois group over  $\mathbb{Q}$ . In this talk I will describe a joint work with Pedro Lemos and Samir Siksek, concerning the realization of  $\text{GSp}_6(\mathbb{F}_\ell)$  as a Galois group for infinitely many odd primes  $\ell$ . Moreover I will describe uniform realizations of linear groups.

### Jacobian varieties of genus 3 and the inverse Galois problem

SARA ARIAS DE REYNA, Universidad de Sevilla

**ABSTRACT:** The inverse Galois problem, first addressed by D. Hilbert in 1892, asks which finite groups occur as the Galois group of a finite Galois extension  $K|\mathbb{Q}$ . This question is encompassed in the general problem of understanding the structure of the absolute Galois group  $G_{\mathbb{Q}}$  of the rational numbers.

A deep fact in arithmetic geometry is that one can attach compatible systems of Galois representations of  $G_{\mathbb{Q}}$  to certain arithmetic-geometric objects (e.g. abelian varieties). These representations can be used to realise classical linear groups as Galois groups over  $\mathbb{Q}$ .

In this talk we will discuss the case of Galois representations attached to Jacobian varieties of genus  $n$  curves. For  $n = 3$ , we provide an explicit construction of curves  $C$  defined over  $\mathbb{Q}$  such that the action

of  $G_{\mathbb{Q}}$  on the group of  $\ell$ -torsion points of the Jacobian of  $C$  provides a Galois realisation of  $\mathrm{GSp}_6(\mathbb{F}_{\ell})$  for a prefixed prime  $\ell$ . This construction is a joint work with Cécile Armana, Valentijn Karemaker, Marusia Rebolledo, Lara Thomas and Núria Vila, and was initiated as a working group in the Conference *Women in Numbers Europe* (CIRM, 2013).

## Iwasawa theory and STNB, a personal view

FRANCESC BARS, Universitat Autònoma de Barcelona

**ABSTRACT:** Iwasawa theory is the study of objects of arithmetic interest over infinite towers of global fields.

Iwasawa explored this study for the  $p$ -th cyclotomic extension of number fields, obtaining a general theory of cyclotomic Iwasawa modules, formulating some sort of cyclotomic Iwasawa main conjecture. This was latter generalized considering  $\mathbb{Z}_p^d$ -extensions by Greenberg and many others (In STNB2014 we worked in a proof for some specific object for the cyclotomic Iwasawa main conjecture).

Kato in 1993 observed a strong relation between Iwasawa main conjecture with the  $p$ -part of Tamagawa number conjecture on  $p$ -valuation of special values of  $L$ -functions for a motive, therefore for attack BSD a good Iwasawa main conjecture for non-abelian infinite towers will be useful, with this inspiration a lot of big mathematicians develope lasts years non-commutative Iwasawa theory for example for extensions of the form  $GL_2(\mathbb{Z}_p)$ . Kato and Fukaya formulated a general non-commutative Iwasawa main conjecture for what knowadays is called  $\Lambda$ -rings, some sort of Iwasawa algebras. (In a talk in STNB2008 we introduced a little bit the non-commutative formulation).

By explicit class field theory, on globals fields of positive characteristic, appears natural to consider abelian profinite groups (in the infinite tower of global fields) which are not  $\Lambda$ -rings by adding the torsion of rank 1 Drinfeld modules which is the "cyclotomic"-tower in the positive characteristic, and one could obtain a non-commutative and non-abelian Iwasawa algebras if one introduces torsion of higher rank Drinfeld modules (considering the work of Pink and his school). (In

STNB2001 we worked on Drinfeld modules and Hayes explicit class field theory, in particular the construction of the "cyclotomic"-tower). First, in the talk we want to remember the classical commutative Iwasawa theory over a global field of positive characteristic, and present the recent results on "Carlitz-cyclotomic" commutative Iwasawa theory (insights on non-noetherian Iwasawa theory began since STNB2010). After, we will try to present few of the results on non-commutative Iwasawa theory of Witte and Burns for  $\Lambda$ -rings (workshops inside STNB2010) and the interplay of Waldhausen categories (from PhD of Witte).

### **On the equation $X^n - 1 = B.Z^n$**

BORIS BARTOLOMÉ & PREDA MIHAILESCU,  
Georg-August-Universität Göttingen

**ABSTRACT:** We consider the Diophantine equation  $X^n - 1 = B.Z^n$ , where  $B \in \mathbb{Z}$  is understood as a parameter. We prove that if this equation has a solution, then either the Euler totient of the radical,  $\varphi(\text{rad}(B))$ , has a common divisor with the exponent  $n$ , or the exponent is a prime and the solution stems from a solution to the diagonal case of the Nagell-Ljunggren equation:  $(X^n - 1)/(X - 1) = n^e.Y^n$ ;  $e \in \{0; 1\}$ . This allows us to apply recent results on this equation to the binary Thue equation in question. In particular, we can then display parametrized families for which the Thue equation has no solution. The first such family was proved by Bennett in his seminal paper on binary Thue equations.

### **Badly approximable numbers in Diophantine Approximation**

PALOMA BENGOCHEA, University of York

**ABSTRACT:** I will talk about two different types of approximation of  $n$ -dimensional real vectors: the approximation by rationals, and the approximation by the integer multiples of an arbitrary fixed real vector. In the first case, we talk about classical Diophantine approximation and in the second case we talk about twisted Diophantine

approximation. The twisted approximation can easily be interpreted in terms of toral rotations. I will define the concept of badly approximable numbers in both types of approximations and will discuss the "size" of the sets of badly approximable numbers in  $\mathbb{R}^n$  and in submanifolds. The problem of determining the size was settled a long time ago for the classical set, whereas it follows from recent results for the twisted set, including recent results by Moshchevitin and myself and Stepanova and myself.

## On the reciprocity law for $p$ -adic Green functions

ALBERTO CÁMARA, University of Nottingham

**ABSTRACT:** The reciprocity law for Green functions (also known as “integrals of differentials of the third kind”) on  $p$ -adic curves was established by Coleman (1989) and Colmez (1998) by means of  $p$ -adic integration. We will discuss a new approach to this reciprocity law by means of  $p$ -adic functional analysis.

## On the Euler system of CM points of Shimura curves

CARLOS DE VERA, Universität Duisburg-Essen

**ABSTRACT:** Kolyvagin’s method for the Euler system of CM points on modular curves has been extended by Nekovář to the setting of Shimura curves, with applications to the BSD conjecture for modular abelian varieties. However, Nekovář’s result is far from being explicit, in the sense that it relies on a given Shimura curve parameterization and on the existence of CM points of a given conductor on the relevant Shimura curve. In a joint work with M. Longo and V. Rotger, we make explicit both of these constructions.

In the case of elliptic curves, our main result can be summarized as follows. Let  $E/\mathbb{Q}$  be an elliptic curve,  $K$  be an imaginary quadratic field,  $\chi$  be a ring class character of conductor  $c$ , and suppose that the functional equation of the twisted  $L$ -series  $L(E/K, \chi, s)$  has sign  $-1$ . Then we construct a (‘minimal’) Shimura curve parametrization

$X \rightarrow E$ , depending only on the data  $(E, K, \chi)$ , such that the set of CM points in  $X(K_c)$  is non-empty, where  $K_c$  denotes the ring class field of conductor  $c$  of  $K$ . By applying Nekovář's result, if  $L'(E/K, \chi, 1) \neq 0$  then  $(E(K_c) \otimes_{\mathbb{Z}} \mathbb{C})^\chi$  has dimension 1, as predicted by BSD.

Joint work with Matteo Longo (Padova) and Victor Rotger (UPC)

## The 2004 STNB on Fontaine-Mazur and Serre's conjectures, and its influence on my proofs of cases of these two conjectures

LUIS DIEULEFAIT, Universitat de Barcelona

**ABSTRACT:** We will recall how in the 2004 STNB we were among the first to emphasize the importance of “potential modularity” (a result by R. Taylor) in connection to the proof of modularity conjectures. My proof of “existence of compatible systems” and of the first cases of the Fontaine-Mazur conjecture for  $\mathrm{GL}(2)$  was already of public domain before 2004, and right after the 2004 STNB I came with the key insight (deduced also independently by C. Khare) that potential modularity (combined with level lowering for Hilbert modular forms and other tools) could also be used to prove “existence of minimal lifts”, a result that combined with my previous results on Fontaine-Mazur gave as a corollary the proof of the first cases of Serre’s conjecture (small level and weight).

## On loci of smooth plane curves with non-trivial automorphism groups

ESLAM FARAG, Universitat Autònoma de Barcelona

**ABSTRACT:** This is a notation: Let  $M_g$  be the moduli space of smooth, genus  $g$  curves over an algebraically closed field  $K$  of zero characteristic. Consider the locus  $M_g^{Pl}$  where

$$M_g^{Pl} := \{\delta \in M_g \mid \exists \text{ a smooth, genus } g \text{ plane model}\}.$$

Now, if  $G$  is a finite non-trivial group then define the loci  $M_g^{Pl}(G)$  and  $\widetilde{M}_g^{Pl}(G)$  as

$$\begin{aligned} M_g^{Pl}(G) &:= \{\delta \in M_g^{Pl} \mid G \cong \text{a subgroup of } \text{Aut}(\delta)\}, \\ \widetilde{M}_g^{Pl}(G) &:= \{\delta \in M_g \mid G \cong \text{Aut}(\delta)\}. \end{aligned}$$

In particular, we have  $\widetilde{M}_g^{Pl}(G) \subseteq M_g^{Pl}(G) \subseteq M_g^{Pl} \subseteq M_g$ .

This talk is devoted to present the results, which have been obtained on these loci. For instance, some aspects on the irreducibility of  $\widetilde{M}_g^{Pl}(G)$  and its interrelation with the existence of “normal forms”, the analogy of Henn’s results on quartic curves, but for degree 5 curves (jointly with Francesc Bars), and also an account on the set of twists of such loci (Jointly with Francesc Bars and Elisa Lorenzo).

## On fields of definition of $\mathbb{Q}$ -curves and Sato-Tate groups of abelian surfaces

FRANCESCA FITÉ, Universität Duisburg-Essen

**ABSTRACT:** Let  $A$  be an abelian surface defined over  $\mathbb{Q}$  that is isogenous over  $\overline{\mathbb{Q}}$  to the square of an elliptic curve  $E$ . If  $E$  does not have complex multiplication (CM), one can deduce from results of Ribet and Elkies, concerning fields of definition of  $\mathbb{Q}$ -curves, that  $E$  admits a model defined over a biquadratic extension of  $\mathbb{Q}$ . We will show that, in our context, one can adapt Ribet’s methods to treat the case in which  $E$  has CM. We find two applications of this analysis to the theory of Sato-Tate groups of abelian surfaces: First, we show that 18 of the 34 existing Sato-Tate groups of abelian surfaces over  $\mathbb{Q}$ , only occur among at most 51  $\overline{\mathbb{Q}}$ -isogeny classes of abelian surfaces over  $\mathbb{Q}$ ; Second, we provide an answer to a question of Serre on the existence of a number field over which abelian surfaces can be defined realizing each of the 52 existing Sato-Tate groups of abelian surfaces over number fields. This is an ongoing project with Xevi Guitart.

## An extension of the Faltings-Serre method

NUNO FREITAS, Universitat de Barcelona

**ABSTRACT:** Faltings showed that a finite amount of computation is enough to decide whether two  $\ell$ -adic representations of the absolute Galois group of a number field  $K$  into  $GL_n(\mathbb{Z}_p)$  are isomorphic. Serre turned this into a practical algorithm and when  $n = p = 2$  applied it to show there is only one isogeny class of elliptic curves of conductor 11. Later Dieulefait–Guerberoff–Pacetti fully automatized and implemented Serre’s ideas and applied it to prove modularity of many elliptic curves over imaginary quadratic fields.

In this talk we will discuss possible extensions of the Faltings-Serre method like working with  $p > 3$  or  $n = 4$ , motivated by applications to modularity of abelian surfaces. This is ongoing work with Lassina Dembele and Luis Dieulefait.

## Jacobians of Mumford curves, a new perspective for generalizations

IAGO GINÉ, Universitat Autònoma de Barcelona

**ABSTRACT:** Jacobians of Mumford curves are classical objects well known over  $p$ -adic fields since the book on Schottky groups and theta functions by Gerritzen and van der Put and the paper by Manin and Drinfeld. More recently, Darmon, Longi and Dasgupta, between others, gave a new construction by means of multiplicative integrals. We use this new approach with the tools of Berkovich analytic theory to remake the construction over any complete non-archimedean field.

## Fermat i el seu darrer teorema

JOSEP GONZÁLEZ, Universitat Politècnica de Catalunya

**ABSTRACT:** És una xerrada de caire històric. A través de la correspondència de Fermat, estudiem la seva evolució en el camp de l’Aritmètica per esbrinar què (i com) havia demostrat Fermat del seu darrer Teorema.

## **Al-karají y yo**

JOAN CARLES LARIO, Universitat Politècnica de Catalunya

**ABSTRACT:** El matemàtic i astrònom persa Al-karají va fer néixer les varietats abelianes al segle X en l'islam medieval. Jo explicaré per què els noms de Fibonacci, Fermat, Nagell, Weber, Heegner, Birch, Bayer, Gross, Shimura, Purkait, entre d'altres, em remunten a Al-karají.

## **Twists of the Klein quartic: a complete classification and a modular interpretation**

ELISA LORENZO, Universiteit Leiden

**ABSTRACT:** The Klein quartic is, up to isomorphism, the genus 3 curve with biggest automorphism group. We will show a classification of its twists over number fields, which provides a complete classification of twists of non-hyperelliptic genus 3 curves defined over number fields, already started by the speaker in her thesis. The Klein quartic is isomorphic to the modular curve  $X(7)$ , which endows the twists with a modular interpretation. We use this interpretation to provide counterexamples to the Hasse principle.

## **On the Iwasawa-Leopoldt Conjecture**

P. MIHAILESCU, Georg-August-Universität Göttingen

**ABSTRACT:** The structure of the  $p$ -part  $A$  of the  $p$ -th cyclotomic field, simple as it should be, is poorly understood. The guiding expectation is the Kummer-Vandiver conjecture, which asserts that  $A^+ = 1$ . However, computations suggest even stronger restriction: the exponent of  $A$  should be  $p$  and, in the Iwasawa tower,  $A_\infty$  should have linear annihilators. On the other side, Iwasawa and Leopoldt proposed a slightly weaker conjecture than Kummer-Vandiver, which stipulates that  $A$  is  $\mathbb{Z}_p[G]$  cyclic, with  $G$  the Galois group. In this talk we discuss the implications of the various facts and conjectures and prove the following result: Assuming the Greenberg conjecture,  $A^+$  is  $\mathbb{Z}_p[G]$ -cyclic.

## Uniformització hiperbòlica d'algunes corbes de Shimura en el cas $D = 22$

JOAN NUALART, Universitat de Barcelona

**ABSTRACT:** En aquesta xerrada determinarem funcions automorfes explícites que donen els models canònics de la corba  $X(22, 1)$  i d'alguns quocients i recobriments seus. Començarem estudiant com es pot solucionar aquest mateix problema en el cas  $D = 6$  i introduirem les eines necessàries per a estendre'l al nostre cas (que no es redueix a un cas triangular). Finalment veurem algunes aplicacions d'aquestes tècniques, per exemple, per al càlcul de desenvolupaments d'aquestes funcions automorfes al voltant de punts de multiplicació complexa arbitraris.

## Confluencia formal de operadores diferenciales cuánticos

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**ABSTRACT:** El proceso de confluencia consiste en reemplazar “diferencial” por “en diferencias finitas” o “en  $q$ -diferencias” en una ecuación diferencial, para intentar deducir información sobre las soluciones de la ecuación original a partir de soluciones de ecuaciones en diferencias o en  $q$ -diferencias. De hecho los tres tipos de ecuaciones se pueden ver como casos particulares de ecuaciones diferenciales *twistadas* [1].

Explicaremos cómo se pueden definir, a la Grothendieck-Bethelot, operadores diferenciales twistados de nivel infinito y demostraremos que, formalmente, estos objetos son independientes del *twist*. De esto se puede deducir que los operadores diferenciales usuales son formalmente límites de operadores diferenciales cuánticos [2].

Nuestro método se aplica también en característica positiva y, lo que nos interesa más, en  $q$ -característica positiva, es decir, cuando  $q$  es una raíz  $p$ -ésima primitiva de la unidad. En ese caso necesitamos, como explicaremos, completar a una familia de raíces  $p^n$ -ésimas de  $q$ . El objetivo final de esta línea de trabajo es intentar entender la correspondencia de Simpson  $p$ -ádica como límite de correspondencias de

Simpson cuánticas [3].

Muchas de nuestras construcciones están inspiradas en las que se usan en cohomología cristalina. Seguro que los participantes en los STNB de 1999 y 2000, celebrados en Collbató, recuerdan todos los detalles, pero el resto de los asistentes no deben preocuparse porque no será necesario conocerlos.

## Referencias

- [1] Bernard Le Stum and Adolfo Quirós.  
Twisted calculus. *arXiv:1503.05022*, 2015.
- [2] Bernard Le Stum and Adolfo Quirós. Formal confluence of quantum differential operators. *arXiv:1505.07258*, 2015.
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## Hopf Galois and ramification

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**ABSTRACT:** Una bona part de la Teoria de Nombres té a veure amb l'estructura dels cossos de nombres i de les extensions de cossos de nombres. Quan considerem una extensió  $L/K$ , com més simètric és  $L$  en relació a  $K$ , més fàcil és entendre'l. Per això normalment ens limitem a l'estudi d'extensions galoisianes. En aquesta situació, el Teorema de la Base Normal proporciona una base formada per una sola òrbita galoisiana. La qüestió es pot refinar per preguntar-nos sobre l'existència d'una base normal entera. El teorema de Noether ho resol establint l'obstacle de la ramificació salvatge.

A la vista d'aquest resultat podem mirar de generalitzar considerant extensions no galoisianes, però encara amb una certa "simetria", com passa amb les extensions Hopf Galois, o bé considerant per a una extensió galoisiana la informació addicional que puguin aportar les diverses estructures Hopf Galois existents. En qualsevol cas, es tracta de veure si aquestes estructures ens permeten acostar-nos al tractament de les extensions salvatgement ramificades.

**Aritmètica en corbes de Picard amb Multiplicació Complexa**

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**ABSTRACT:** Presentem una implementació en Sage per al càlcul de les classes d'isomorfisme de varietats abelianes principalment polaritzades amb multiplicació complexa per l'anell d'enters d'un cos de Picard (extensió normal i cíclica  $K|\mathbb{Q}$  de grau 6 que conté les arrels cúbiques de la unitat). Aquestes varietats abelianes poden ser vistes com a Jacobianes de corbes de Picard. Fem córrer l'algoritme per a tots els cossos de Picard amb nombre de classes  $\leq 11$ .

**Configurations of points and circles on surfaces  
from uniform dessin d'enfants**

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**ABSTRACT:** Geometric realizations of configurations is a classical subject in geometry. We show how to find geometric realizations of interesting configurations on surfaces in terms of points and isometric circles, by defining them as neighbourhood geometries of uniform dessin d'enfants. Several examples are discussed.

Joint work with Milagros Izquierdo, Linköping University, Sweden







