Seminari de Teoria de Nombres (UB-UAB-UPC)
STNB2017, 31a ed.

Abstracts of the STNB2017

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Organising Committee: F. Bars, X. Guitart, B. Plans, A. Travesa

Introduction

The Seminari de Teoria de Nombres (UB-UAB-UPC) is a group of researchers in number theory mainly from Universitat de Barcelona, Universitat Autònoma de Barcelona and Universitat Politècnica de Catalunya.

Since 1986, this research group has been organizing activities as courses, workshops and conferences in number theory in our country. The main annual activity is known as Seminari de Teoria de Nombres de Barcelona, joining researchers in this area in Barcelona metropolitan area and attracting people from other places and foreign countries too. In 2017 we are celebrating its 31th edition.

The two learning courses of this edition of the Seminari de Teoria de Nombres de Barcelona (STNB2017) are interconnected. In the first course, the aim is to present the foundations of Hida families and has been coordinated by Santiago Molina. In the second one, applications that this theory has found in the theory of elliptic curves are provided, and has been coordinated by Francesc Fité. Moreover in the STNB2017 there are several contributed talks by participants.

This booklet contains the abstracts of all the scheduled talks as provided by the respective speakers or by the coordinators of the learning courses.

We hope you will find this information useful.

Barcelona, January 2017

F. Bars, X. Guitart, B. Plans, A. Travesa

Contents

Hida families and applications

In this edition of the SNTB, the two learning courses of the seminar are interconnected. In the first course, the aim will be to present the foundations of Hida families. In the second one, we will give applications that this theory has found in the theory of elliptic curves.

The first example of a p-adic family of modular forms was the Eisenstein family, considered by Serre in [Ser72]. The theory took off with the seminal works of Hida (see [Hid86a], [Hid86b]), where he constructed p-adic families of cuspforms (the so-called *Hida families*), varying continously with the weight, which are simultaneous eigenforms of the Hecke operators. Hida's construction had a certain limitation: only *p-ordinary* Hecke eigenforms of level coprime to p belong to a Hida family. Coleman and Mazur constructed a p-adic rigid anlaytic curve containing any eigenform of level coprime to p. The goal of the first course is to review all these classical constructions.

We will start the second course by presenting the p-adic Birch and Swinnerton-Dyer conjecture. It predicts the existence of an exceptional zero at the critical point of the p-adic L-function attached to an elliptic curve with split multiplicative reduction at p. A formula for the first derivative of the p-adic L-function (the so-called *exceptional zero conjecture*), which implies the p-adic BSD conjecture in p-adic rank 1, was experimentally observed by Mazur, Tate and Teitelbaum [MTT86], and proven by [GS93], using as a fundamental tool the theory of Hida families. The course will proceed by presenting results concerning derivatives of p-adic L-functions, whose proofs also make use of the theory of Hida families and whose applications range from the rationality of Stark–Heegner points to the theory of special values of L-functions.

Abstracts for: Hida families

(learning course)
Coordinator: Santiago Molina

The space of ordinary forms

Santiago Molina
Centre de Recerca Matemàtica

The goal of this talk is to provide an introduction for the future talks. In this talk we will see the space of ordinary modular forms from different points of view. We will discuss the different perceptions of an ordinary modular form as power series, as a modular symbol or as a section in certain sheaf. Later talks will put such perceptions of an ordinary modular form into a p-adic family. Main reference: [Laf].

The space of Λ-adic modular forms and the Eisenstein family

Xavier Guitart
Universitat de Barcelona

We explain [Laf, §4.1] and [Laf, §4.2]. More concretely, first define the space of Λ-adic modular forms. Then, as an example of Λ-adic modular form, present the Eisenstein family. For this, you will require the existence of the p-adic L-function attached to a Dirichlet character. Show in detail its existence following the notes [Gui]. This is a basic construction. In the talk of Óscar Rivero entitled p-adic analog of BSD conjecture, we will attach p-adic L-functions to elliptic curves. Main reference: [Laf].

The structure of the space of Λ-adic modular forms

Francesc Fité
Universitat Politècnica de Catalunya

We explain from [Laf, §4.3] to [Laf, §4.6]. In particular, we show that the the space of Λ-adic modular forms is free and of finite rank over Λ. We will explain also that modular forms can be lifted to Λ-adic ones. In order to do that we will have to explain Theorem 3.2.1 of [Laf] first. We will also explain the ordinary Hecke algebra and specialization (as in [Hid86b]). Main references: [Laf], [Hid86b].

Overconvergent modular symbols and the cuspidal eigencurve

Chris Williams
Imperial College London

With this talk, we aim to introduce Stevens' construction of the Eigencurve. We will observe the eigenspace defined by a convergent modular eigenform in a space of modular symbols. We will show how to put such overconvergent modular symbols into families. Main references: [DHHJPR16], [Bel].

The eigencurve: geometric construction

Adel Betina
Universitat Politècnica de Catalunya

The aim of this talk is to give a gentle but fairly complete introduction to the Coleman-Mazur eigencurve. I will begin by recall the construction of V.Pilloni of the overconvergent modular sheave using coherent cohomology. After I will give a short introduction about the p-adic spectral theory of the Atkin operator U_p and how we get the Coleman-Mazur eigencurve. I will finish this talk by giving some geometric properties of the Coleman-Mazur Eigencurve. Main references: [CM96], [Pil13].

Abstracts for: p-adic methods for elliptic curves

(learning course)
Coordinator: FRANCESC FITÉ

p-adic analog of BSD conjecture

ÓSCAR RIVERO
Universitat Politècnica de Catalunya

This talk does not involve Hida families, but will present the basic type of question that we want to look at in this course, with main reference [MTT86]. Construction of the Mazur–Swinnerton-Dyer p-adic L-function attached to a p-allowable stabilized eigenform, §I.13 (announce the theorem of Vishik, Amice-Vélu §I.11). Present the functional equation, §II.17. Formulate the p-adic BSD conjecture, describing each of the terms appearing in the conjectural formula for the leading term of the p-adic L-function, §II.10. That is, introduce the canonical cyclotomic pairing and the Schneider p-adic pairing (this will be needed in Talk entitled *The Mazur-Tate-Teitelbaum conjecture in the rank one setting* by Victor Rotger). Formulate the exceptional zero conjecture, §II. 13 (i.e. the formula for the first derivative of the p-adic L-function). Its proof will be the topic of the talk entitled *The exceptional zero conjecture* by Daniel Barrera.

The exceptional zero conjecture

DANIEL BARRERA
Universitat Politècnica de Catalunya

The goal of this talk is to prove [GS93, Thm. 0.3]. Relate this theorem to the p-adic BSD conjecture in p-adic rank 1. The main reference is [GS93]. The talk will skip §1,§2, whose content will have been covered in the talk entitled *The structure of the space of* Λ*-adic forms* by Francesc Fité, and §4, which will have been expained in the talk entitled *p-adic analog of BSD-conjecture* by Óscar Rivero. The talk will focus on Thm. 3.18 of §3 and Thm. 5.15 of §5. Introduce in detail the 2-variable Mazur–Kitagawa p-adic L-function, which will play a fundamental role in the following talks. Combine Thm. 3.18 and Thm. 5.15 to deduce the exceptional zero conjecture as in §7 (for this, you may simply consider the case $E = X_0(11)$ as in [GS93, Introduction] if you judge it convenient).

A second derivative result for p-adic L-function

Carlos de Vera
Universität Duisburg-Essen

The aim of this talk is to discuss [BD07a, Thm. 1]. From the talk entitled *The exceptional zero conjecture* by Barrera, you may assume familiarity with the Mazur–Kitagawa p-adic L-function and its functional equation. In the setting of [BD07a], start by presenting the dichotomy coming from the sign in this functional equation. Focus this talk on the $+1$ case, by mentioning that the -1 corresponds to the situation of the talk *The exceptional zero conjecture*. We skip §1 and §2, although you should recall the notations introduced in these sections when necessary. Introduce the 2-variable p-adic L-function of [BD07a, Def. 3.5] associated to a Hida Family and a quadratic imaginary field, and present its interpolation property (Thm. 3.8 and 3.12). From this easily deduce its factorization in terms of two Mazur–Kitagawa p-adic L-functions. Evoke the analogy with the key factorization of the citated talk *The exceptional zero conjecture*. Conclude the proof of the main result as in §5.2. Main reference: [BD07a].

The Mazur-Tate-Teitelbaum conjecture in the rank one setting

VICTOR ROTGER
Universitat Politècnica de Catalunya

We shall explain Rodolfo Venerucci's results on the Mazur-Tate-Teitelbaum exceptional zero conjecture for the p-adic L-function of an elliptic curve $E/\mathbb{Q}$ of Mordell-Weil rank 1. The key point of the argument is the interplay between the Euler systems of Heegner points and Beilinson-Kato elements. A fundamental tool employed in the proof is the theory of p-adic variation of modular forms, special values of L-functions and logarithms of cycle classes. Main reference: [Ven16].

The rationality of traces of Stark-Heegner points

MARC MASDEU
University of Warwick

Let $E/\mathbb{Q}$ be an elliptic curve over the rationals, and let $K/\mathbb{Q}$ be a real quadratic field for which $L(E/K, s)$ vanishes to odd order at $s = 1$. Let p be a prime of multiplicative reduction. Darmon attached to each order $\mathcal{O}$ of K a set of points P_i in $E(K_p)$ (as many as the class number of $\mathcal{O}$), and conjectured that each of them was in fact defined over the ring class field of $\mathcal{O}$. The goal of this talk is to define these so-called Stark–Heegner points, and to discuss (a particular case of) the main result of [BD09] to prove, under the additional assumption that E has *split* multiplicative reduction at p, that the sum of these points is defined over K (in particular, it is algebraic), and that it is nontorsion if and only if $L'(E/K, 1)$ is nonzero. The alluded main result of [BD09] heavily uses the results of [BD07a] discussed in the talk entitled *A second derivative result for p-adic L-functions* by Carlos de Vera. Main reference: [BD09].

References for Hida Families and applications

[Bel] J. Bellaïche, *Eigenvarieties, families of Galois representations, p-adic L-functions.*

[BC09] J. Bellaïche, G. Chenevier, *Families of Galois representations and Selmer groups*, Astŕisque No. **324** (2009)

[BD07a] M. Bertolini, H. Darmon, *Hida families and rational points on elliptic curves*, Invent. Math. **168** (2007), no. 2, 371–431.

[BD07b] M. Bertolini, H. Darmon, *The p-adic L-functions of modular elliptic curves*, Mathematics Unlimited – 2001 and Beyond, Springer Verlag (2001) 109–170.

[BD09] M. Bertolini, H. Darmon, *The rationality of Stark-Heegner points over genus fields of real quadratic fields*, Annals of Mathematics **170** (2009) 343-369.

[CM96] R.F. Coleman, B. Mazur, *The eigencurve*, Galois representations in arithmetic algebraic grometry (Durham, 1996) (A.J. Scholl, R.L Taylor eds.), London Math. Soc. Lecture Notes Series 254, Cambridge Univ. Press, 1998, 1–113.

[DLR15] H. Darmon, A. Lauder, V. Rotger, *Stark points and p-adic iterated integrals attached to modular forms of weight one*, Forum of Mathematics, Pi (2015), Vol. 3, e8, 95 pages.

[DHHJPR16] E.P. Dummit, M. Hablicsek, R. Harron, L. Jain, R. Pollack, D. Ross, *Explicit computations of Hida families via overconvergent modular symbols*, to appear in Research in Number Theory, 2016.

[Eme09] M. Emerton, *p-adic families of modular forms (after Hida, Coleman, and Mazur)*, Séminaire Bourbaki, 62ème année, 2009-2010, no. 1013.

[GS93] R. Greenberg, G. Stevens, *p-adic L-functions and p-adic periods of modular forms*, Invent. Math. **111**(2), (1993), 407–447.

[Gui] X. Guitart, *Mazur's construction of the Kubota–Leopold p-adic L-function*, available on Xevi Guitart's web page.

[Hid86a] H. Hida, *Iwasawa modules attached to congruences of cusp forms*, Ann. Sci. École Norm. Sup. **19** (1986), 231–273.

[Hid86b] H. Hida, *Galois representations into* $GL_2(\mathbb{Z}_p[[X]])$ *attached to ordinary cusp forms*, invent. math. **85**, 545–613 (1986).

[Laf] M.J. Lafferty, *Hida Theory.*

[MTT86] B. Mazur, J. Tate, J. Teitelbaum, *On p-adic analogues of the conjectures of Birch and Swinnerton-Dyer*, Invent. Math. **84** (1) (1986), 1–48.

[Pil13] V. Pilloni, *Overconvergent modular forms*, Ann. Inst. Fourier (Grenoble) **63** (2013), no. 1, 219–239.

[Rub92] K. Rubin, *p-adic L-functions and rational points on elliptic curves with complex multiplication*, Invent. Math. **84** (1) (1986), 323–350.

[Ser72] J.-P. Serre, *Formes modulaires et fonctions zêta p-adiques*, Modular functions of one variable, III (Proc. Internat. Summer School, Univ. Antwerp, 1972), Springer Lecture Notes **350**, 191–268.

[Ven16] R. Venerucci, *Exceptional zero formulae and a conjecture of Perrin–Riou*, Invent. Math. **203** (3) (2016), 923-–972.

Abstracts for the contributed talks

(in alphabetical order of authors)
Coordinator: B. Plans

Galois representations with values in mod p Hecke algebras

Laia Amorós
Université du Luxembourg

I will explain how one can construct Galois representations that take values in local mod p Hecke $\mathbb{F}_q$-algebras. I will then show how one can compute explicitly the image of such a Galois representation assuming that the corresponding residual representation has big image. Finally I will show some computations of these images in concrete examples.

On Maeda's Conjecture

Paloma Bengoechea
ETH Zürich

Let $T_{n,k}(X)$ be the characteristic polynomial of the n-th Hecke operator acting on the space of cusp forms of weight k. A popular conjecture, called Maeda's Conjecture, asserts that, for every n, $T_{n,k}(X)$ is irreducible in $\mathbb{Q}[X]$ and has full Galois group over $\mathbb{Q}$. We will discuss some recent progress on the conjecture and will show the following: Let k be any positive integer congruent to 0 modulo 4. If one $T_{n,k}(X)$ is irreducible and has full Galois group, then the same is true of $T_{p,k}(X)$ for all primes p.

CM cycles on Kuga-Sato varieties over Shimura curves and Selmer groups

Carlos de Vera
Universität Duisburg–Essen

Let f be a normalized Hecke eigenform of even weight ≥ 4 and level N, and let K be an imaginary quadratic field. Under the Heegner hypothesis that every prime dividing N splits in K, Nekovář extended Kolyvagin's method of Euler systems to this higher weight situation by replacing CM points on modular curves by certain algebraic (CM) cycles on the Kuga–Sato variety on which the Galois representation associated to f is realized. As a consequence, he was able to bound the Selmer group of f under the assumption that certain cohomology class in the bottom layer of the Euler system does not vanish.

In a recent joint work with Y. Elias, we have adapted Nekovář's result to the scenario where the Heegner hypothesis is relaxed in such a way that one moves to Shimura curves. The Galois representation attached to f can be realized as well in the cohomology of a self-fold fiber product of the universal abelian surface over a Shimura curve, where one can still construct a family of algebraic cycles leading to an Euler system. I will sketch how to construct such cycles in this setting, and explain their main properties giving rise to the desired Euler system to which one can apply Kolyvagin's machinery.

A multi-Frey approach to some Fermat type equations

Nuno Freitas
University of British Columbia

In this talk I will report on ongoing work with Nicolas Billerey, Imin Chen and Luis Dieulefait. We focus on Fermat equations of the form $x^r + y^r = dz^p$, where r is a fixed prime, via the multi-Frey technique.

The first step to apply the modular method to the equation above is to attach a Frey curve to a putative solution of it. It is known that if we can find multiple Frey curves then maybe we can obtain sharper bounds on the exponent p for which the equation has no solutions satisfying $|xyz| > 1$.

We will discuss other advantages of having multiple Frey curves. Namely, how we can "transfer" irreducibility of the mod p Galois representations from one Frey curve to another and how we can reduce the amount of computations of modular forms required (which sometimes are too large to be feasible).

We will sketch how these ideas can be used to attack the equation above in the cases $r = 5, 13$ and $d = 3$.

Asymptotic distribution of Hecke points over $\mathbb{C}_p$

Sebastián Herrero
Chalmers University of Technology and University of Gothenburg

In this talk I will present some results about the asymptotic distribution of Hecke points on the moduli space of elliptic curves over $\mathbb{C}_p$. These points correspond to elliptic curves which admit an isogeny of a given degree to a fixed elliptic curve. For this we use Tate's uniformization theory of elliptic curves with bad reduction and the existence of canonical subgroups for ordinary and not too supersingular elliptic curves, among other results. Our techniques also apply to the study of the asymptotic distribution of CM points over $\mathbb{C}_p$. This is joint work with Ricardo Menares (P. Universidad Católica de Valparaíso) and Juan Rivera-Letelier (University of Rochester).

On Selmer Groups and Factoring p-adic L-functions

Enis Kaya
Koç University

S. Dasgupta has proved a formula factoring a certain restriction of a 3-variable Rankin-Selberg p-adic L-function as a product of a 2-variable p-adic L-function related to the adjoint representation of a Hida family and a Kubota-Leopoldt p-adic L-function. Then B. Palvannan proved a result involving Selmer groups that along with Dasgupta's result is consistent with the main conjectures associated to the Galois representations. Under certain additional hypotheses, he also indicated how one can use work of E. Urban to deduce main conjectures for the 3-dimensional representation and the 4-dimensional representation. In this talk, I will give a gentle overview of Palvannan's result.

Lattice points in elliptic paraboloids

Carlos Pastor
ICMAT

Given a compact subset $\mathcal{K}$ of $\mathbb{R}^d$, let $\mathcal{N}(R)$ denote the number of points in the lattice $\mathbb{Z}^d$ that lie within $\mathcal{K}$ after being dilated by a factor $R > 1$. The lattice point problem associated to $\mathcal{K}$ consists in approximating $\mathcal{N}(R)$ for large R, usually in the form $\mathcal{N}(R) = |\mathcal{K}|R^d + O(R^\alpha)$ where $|\mathcal{K}|$ stands for the volume of $\mathcal{K}$. Determining how small the exponent α can be taken in this asymptotic formula is an open problem even for simple regions such as the circle (Gauss problem) or rational ellipsoids in the three-dimensional space. In this talk I will present a recent joint work with F. Chamizo, showing that α may be taken arbitrarily close to $d-2$ for a wide family of truncated elliptic paraboloids in at least three dimensions, and certainly not smaller than $d-2$ in some cases. To do this the problem is first reduced to the estimation of a 2-dimensional quadratic exponential sum, which is then bounded using a simplified version of the circle method.

Stark's conjectures and generalized Kato classes

Óscar Rivero
Universitat Politècnica de Catalunya

During the last years, a great progress in the study of BSD and Block-Kato conjectures has been made. Here, we are interested in two different aspects: on the one hand, the conjecture suggested by Darmon, Lauder and Rotger relating the value of a p-adic iterated integral (whose value may be encoded in Harris-Kudla triple p-adic L-functions) with Stark's units and the value of regulators defined in terms of p-adic logarithms of units in number fields (this can also be formulated for points over elliptic curves and gives interesting results for BSD in analytic rank 2). On the other hand, several works of Bertolini, Darmon, Rotger and others allow us to construct families of cohomology classes satisfying good properties and related with special values of the p-adic L-functions. These classes are obtained via the image through étale and syntomic regulators of distinguished cycles in certain algebraic varieties. In our work, we use the construction of families of cohomology classes interpolating two cuspidal forms to formulate a conjecture about the good behavior of these Kato classes and their relation with Ohta periods, that would imply the main result suggested in the paper of Darmon, Lauder and Rotger.

Algorisme de determinació d'estructures Hopf Galois i les propietats

Marta Salguero
Universitat de Barcelona

La teoria Hopf Galois és una generalització de la teoria de Galois. Donada una extensió de Galois, l'acció del grup de Galois s'estén

a una acció de l'àlgebra de grup. Aquest fet inspira el concepte d'estructura Hopf Galois donada per l'acció d'una àlgebra de Hopf. En aquesta comunicació presentarem un algorisme implementat amb Magma que determina totes les estructures Hopf Galois de les extensions separables de cossos d'un grau donat i diverses propietats d'aquestes. Prèviament, recordarem la teoria Hopf Galois i la traducció al llenguatge de grups en el cas separable.

Asymptotic Fermat's Last Theorem over Number Fields

Haluk Sengun
University of Sheffield

Assuming two deep but standard reciprocity conjectures from the Langlands Programme, we prove that the asymptotic Fermat's Last Theorem holds for imaginary quadratic fields $\mathbb{Q}(\sqrt{-d})$ with $-d \equiv 2, 3 \pmod 4$. For a general number field K, again assuming standard conjectures, we give a criterion based on the solutions to a certain S-unit equation, which if satisfied implies the asymptotic Fermat's Last Theorem. This is joint work with Samir Siksek.