Seminari de Teoria de Nombres (UB-UAB-UPC)
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Abstracts of the STNB2023

Facultat de Matemàtiques, Universitat de Barcelona
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Organising Committee:
F. Bars, L.V. Dieulefait, B. Plans, A. Travesa

Introduction

The *Seminari de Teoria de Nombres (UB-UAB-UPC)* wants to make explicit a warm welcome to all the participants to the 36th edition of our annual Seminar. The *Seminari* is composed, mainly, of researchers in Number Theory of the Barcelona metropolitan area, mostly of Universitat de Barcelona (UB), Universitat Autònoma de Barcelona (UAB) or Universitat Politècnica de Catalunya (UPC), and attracts people from other places or foreign countries too.

This edition of the Seminari de Teoria de Nombres de Barcelona, STNB2023, will contain several **conferences** or **research talks** exposing works of some participants to this event.

In this edition, coffee breaks or social events will not be **scheduled** although it will be possible to take some coffee between sessions. Moreover, as being more and more usual, the talks will be streamed through a suitable Zoom link to all people interested.

We would specially like to thank all the speakers; without them, this event would not be possible.

This booklet contains the abstracts of the scheduled talks as provided by their respective authors. We hope you will find this information helpful and enjoy the Seminari as much as possible.

Barcelona, January 2023

F. Bars, L.V.Dieulefait, B. Plans, A. Travesa

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Abstracts for contributed conferences or communications

(in alphabetical order of speakers)
Coordinator: Bernat Plans

p-adic interpolation of modular forms of infinite slope

Andrea Conti
Université du Luxembourg
Tuesday, Januuary 24, 11:00-11:50

The absolute Galois group $G_{\mathbb{Q}}$ of the rational numbers encodes the structure of all extensions of $\mathbb{Q}$ generated by solutions of polynomial equations. One way to better understand this complicated group is to study its p-adic representations, many of which can be produced starting with a modular form. Identifying modular forms with points of a p-adic variety allows one to both produce more Galois representations by interpolation, and study them using geometric tools. Nowadays we know how to p-adically interpolate forms that are of finite slope, meaning that they are not in the kernel of a certain Hecke operator. It is an open question whether one can do the same for forms of infinite slope: I will present some progress towards proving that in this setting interpolation is only possible for some exceptional forms.

On the infinitely generated locus of Frobenius algebras of rings of prime characteristic

Alberto Fernández-Boix
Seminari de Teoria de Nombres de Barcelona, U. de Valladolid
Monday, January 23, 12:00-12:50

Let R be a commutative Noetherian ring of prime characteristic p, and let M be an R-module. For each integer $e \geq 0$, we denote

by $\mathrm{End}_{p^e}(M)$ the set of p^e-linear maps of M; that is, $\mathrm{End}_{p^e}(M)$ is made up by abelian group endomorphisms $\phi : M \longrightarrow M$ such that $\phi(rm) = r^{p^e}\phi(m)$ for all $(r, m) \in R \times M$. In this way, one can cook up the so-called Frobenius algebra of M,

$$\mathcal{F}^M := \bigoplus_{e \geq 0} \mathrm{End}_{p^e}(M),$$

where multiplication is given by composition of maps. The following question is, to the best of our knowledge, quite open:

Question 0.1. Let R be a commutative Noetherian ring of prime characteristic p. Is it true that $\overline{W^R} := \{\mathfrak{p} \in \mathrm{Spec}(R) : \mathcal{F}^{E_\mathfrak{p}}$ is finitely generated as a ring over its degree zero piece $\}$, where $E_\mathfrak{p}$ is the injective hull of the residue field of $R_\mathfrak{p}$, is an open set?

The goal of this talk is to study Question 0.1 in some detail; more precisely, let $S = R/I$ a commutative Noetherian ring which is a quotient of a regular Noetherian ring R (that is, locally regular) of prime characteristic p. Under these assumptions, we show that $\overline{W^S}$ is closed under generalization; moreover, when S is a Stanley-Reisner ring, we will show that $\overline{W^S}$ is really an open set in the Zariski topology, and provide an algorithmic and explicit description of the defining ideal of its closed complement. The content of this talk is based on joint work with Danny A. J. Gómez Ramírez and Santiago Zarzuela [BGRZ].

References:

[BGRZ] A. F. Boix, D. A. J. Gómez-Ramírez, and S. Zarzuela. On the infinitely generated locus of Frobenius algebras of rings of prime characteristic. Available at https://arxiv.org/pdf/2203.08511.pdf. 1

On reduction steps for Leopoldt's conjecture

Fabio Ferri
University of Exeter

`Wednesday, January 25, 10:30-11:20`

Let p be a rational prime and let L/K be a Galois extension of number fields with Galois group G. Under some hypotheses, we show

that Leopoldt's conjecture at p for certain proper intermediate fields of L/K implies Leopoldt's conjecture at p for L; a crucial tool will be the theory of idempotent relations in $\mathbb{Q}[G]$. We also consider relations between the Leopoldt defects at p for intermediate extensions of L/K. We finally show that our results combined with some techniques introduced by Buchmann and Sands allow us to find infinite families of nonabelian totally real Galois extension of $\mathbb{Q}$ satisfying Leopoldt's conjecture for certain primes. This is joint work with Henri Johnston.

Local-global principles for quadratic and polyquadratic twists of abelian varieties

Francesc Fité
Universitat de Barcelona
Thursday, January 26, 11:50-12:40

Let A and A' be abelian varieties defined over a number field k. In the talk I will consider the following question: Is it true that A and A' are quadratic twists of one another if and only if they are quadratic twists modulo p for almost every prime p of k? Serre and Ramakrishnan have given a positive answer in the case of elliptic curves and a result of Rajan implies the validity of the principle when the endomorphism ring of A (and A') over an algebraic closure of $\mathbb{Q}$ is just $\mathbb{Z}$. More in general, I will show that the answer is affirmative up to dimension 3, but that it becomes negative in dimension 4. The proof builds on Rajan's result and uses a Tate module tensor decomposition of geometrically isotypic abelian varieties obtained jointly with X. Guitart. I will also discuss a similar result concerning polyquadratic twists of abelian varieties obtained in collaboration with Antonella Perucca.

Linear quadratic Chabauty for integral points on even degree hyperelliptic curves

Stevan Gajović
Max Planck Institute for Mathematics, Bonn
Friday, January 27, 10:30-11:20

The method of Chabauty and Coleman is a powerful method to determine rational points on curves which satisfy $r < g$, where g is the genus of the curve and r is the Mordell-Weil rank of its Jacobian over $\mathbb{Q}$. However, the original method crucially depends on the condition $r < g$. In this talk, we explain how we can use (Coleman-Gross) p-adic heights to obtain an analogous method to compute the integral points on certain even degree hyperelliptic curves satisfying $r = g$. We discuss extensions over (quadratic) number fields. This is joint work with Steffen Müller.

Continued fractions and module structure of extensions of p-adic fields

Daniel Gil
Charles University, Prague

`Thursday, January 26, 10:30-11:20`

Classical Galois module theory has its starting point in the normal basis theorem: every Galois extension posseses a normal basis: a basis consisting in the Galois orbit of some element in the top field. Rings of integers of extensions of local or global fields are not so well behaved, as they need not admit a normal integral basis in general. In fact, Noether [Noe32] showed that a Galois extension of local fields posseses a normal integral basis if and only if it is tamely ramified. However, when the ring of integers is free over an object depending on the Galois group, its associated order, one can construct an integral basis similar to a normal one. This approach was introduced by Leopoldt [Leo59] and it motivated subsequent research on the Galois module structure of wildly ramified extensions. Among these works, F. Bertrandias, J.P. Bertrandias and M.J. Ferton [BF72; BBF72] solved completely the problem for cyclic degree p extensions of p-adic fields, finding criteria for the freeness over the associated order in terms of the continued fraction expansion of the ramification number of the extension over the prime p. However, the proof of this surprising connection is barely sketched and has remained mysterious for decades. In this talk, we review the techniques leading to the proof of their result and present an adaptation to degree p extensions

with dihedral normal closure, for which we use the setting provided by Hopf-Galois theory.

References

[BBF72] Françoise Bertrandias, Jean Paul Bertrandias, and Marie-Josée Ferton. Sur l'anneau des entiers d'une extension cyclique de degré premier d'un corps local (II). In: C.R.Acad.Sc., Paris 274.18 [1972], pp. 1388-1391.

[BF72] Françoise Bertrandias and Marie-Josée Ferton. Sur l'anneau des entiers d'une extension cyclique de degré premier d'un corps local (I). In: C.R.Acad.Sc., Paris 274.18 [1972], pp. 1330-1333.

[Leo59] Heinrich-Wolfgang Leopoldt. Über die Hauptordnung der ganzen Elemente eines abelschen Zahlkörpers. In: Journal für die reine und angewandte Mathematik 1959.201 [1959], pp. 119?149. doi:10.1515/crll.1959.201.119.

[Noe32] Emmy Noether. Normalbasis bei Körpern ohne höhere Verzweigung. In: 1932.167 [1932], pp. 147?152. doi:10.1515/crll.1932.167.147.

Parametrizations of isogeny-torsion graphs of elliptic curves over $\mathbb{Q}$

Enrique González-Jiménez
Universidad Autónoma de Madrid

`Friday, January 27, 11:50-12:15`

The isogeny graph of a $\mathbb{Q}$-isogeny class of elliptic curves defined over $\mathbb{Q}$ consists in a vertex for each elliptic curve in the isogeny class and an edge for each rational isogeny of prime degree between elliptic curves in the isogeny class, with the degree recorded as a label on the edge. The isogeny graphs of elliptic curves over $\mathbb{Q}$ first appeared in the so-called Antwerp tables. Although the first proof (in press) seems to be due to Chiloyan and Lozano-Robledo in 2021. In the first part of this talk we will show parametrizations of these isogeny graphs. Moreover, Chiloyan and Lozano-Robledo define isogeny-torsion graph to be an isogeny graph where, in addition, each vertex is labeled with the abstract group structure of the torsion subgroup of the corresponding

elliptic curve. They classify all the possible isogeny-torsion graphs that occur for $\mathbb{Q}$-isogeny classes of elliptic curves defined over $\mathbb{Q}$. In the last part of this talk we will show parametrizations of these isogeny-torsion graphs.

Distribution of Manin-Stevens elliptic curves in twisted isogeny classes over $\mathbb{Q}$

Joan-Carles Lario
Universitat Politècnica de Catalunya
`Friday, January 27, 12:20-13:10`

Let $\mathcal{A}$ be the $\mathbb{Q}$-isogeny class of an elliptic curve A defined over $\mathbb{Q}$. Does $\mathcal{A}$ contain any distinguished elliptic curve? First, Mazur and Swinnerton-Dyer proposed the so-called strong curve $A_0 \in \mathcal{A}$ which is a Γ_0-optimal quotient of the Jacobian of the modular curve $X_0(M)$, where M is the conductor of A. Later, Stevens suggested that it is better to consider the elliptic curve $A_1 \in \mathcal{A}$ such that it is a Γ_1-optimal quotient of the Jacobian of the modular curve $X_1(M)$. In both cases the Manin constant plays a role, and the Stevens proposal seems to be more intrinsically arithmetic due to the intervention of Néron models, étale isogenies, and Parshin-Faltings heights of the elliptic curves involved in the isogeny class. The Manin-Stevens curve A_1 is the one with minimal height.

Let $G(\mathcal{A})$ be the natural graph attached to the isogeny class $\mathcal{A}$: a vertex for every elliptic curve $A \in \mathcal{A}$, and edges correspond to isogenies of prime degree among them. For every square-free integer d, we can consider the graph $G(\mathcal{A}^d)$ attached to the twisted elliptic curve A^d by the quadratic character of $\mathbb{Q}(\sqrt{d})$. It turns out that $G(\mathcal{A})$ and $G(\mathcal{A}^d)$ are canonically isomorphic as abstract graphs (the isomorphism identifies the vertices with equal j-invariant.)

In this talk we shall discuss the probability distribution of a vertex in $G(\mathcal{A}^d)$ to be a Manin-Stevens elliptic curve as $|d|$ grows to infinity.

(This is work in progress in collaboration with Enrique González-Jiménez, UAM).

Arbres valoratius sobre cossos valorats

Enric Nart
Universitat Autònoma de Barcelona
Monday, January 23 , 10:40-11:30

Fixada una valoració v sobre un cos K, es descriu l'arbre que formen totes les extensions de v al cos $K(x)$ de funcions racionals.

Monogeneïtat de cossos cúbics mitjançant corbes el·líptiques

Francesc Pedret
Universitat Politècnica de Catalunya
Tuesday, January 24, 10:30-10:50

Determinar si un cos de nombres cúbic és monògen és equivalent a resoldre l'equació $|I(X,Y)| = 1$, on $I(X,Y)$ és la forma índex del cos. A aquesta equació li podem asociar una corba de gènere 1 donada per $I(X,Y) = Z^3$. Analitzarem les Jacobianes d'aquestes corbes per a determinar alguns resultats sobre la monogeneïtat de certes extensions cúbiques.

Venkatesh's conjecture for modular forms of weight 1

Víctor Rotger
Universitat Politècnica de Catalunya
Tuesday, January 24, 12:20-13:10

En aquesta xerrada explicaré les conjectures de Venkatesh per a varietats aritmètiques. Ens centrarem en el cas particular de corbes modulars, on Harris i Venkatesh van donar-ne una formulació explícita. Descriuré els resultats obtinguts amb Darmon, Harris, Venkatesh on demostrem la conjectura per a formes modulars de pes 1 de tipus dihedral.

Fundamental domains for the Bruhat-Tits tree for $\mathbf{GL}_2(\mathbb{F}_p)$

Eloi Torrents
Universitat Autònoma de Barcelona
`Thursday, January 26, 12:50-13:10`

The computation of fundamental domains of the Bruhat-Tits tree by the action of quaternionic groups allows the computation of harmonic cocycles on it. These are related to automorphic forms and from this fact are derived several applications, as for example the computation of points on Shimura curves and Heegner points on elliptic curves as done by M. Greenberg and later generalized by C. Franc and M. Masdeu. In this talk we will review these concepts, and we will explain how to apply them in the computation of Heegner points on elliptic curves in cases where the Heegner hypothesis is not satisfied, and therefore the classical arquimedian construction of these points is difficult to compute.

Grups de components de models de Néron

Xavier Xarles
Universitat Autònoma de Barcelona
`Wednesday, January 25, 11:50-12:40`

We will explain how to prove a bound for the coinvariants of a group acting on a finitely generated abelian group, and how this is applied to obtain some results concerning the group of components of the Néron model for tori and for abelian varieties. This result solves a question of Bas Edixhoven from 1995.

Coincidences of Division Fields of an elliptic curve defined over a number field

Zoé Yvon
Université Aix-Marseille
`Wednesday, January 25, 12:50-13:10`

Let E/F an elliptic curve over a number field F. For a positive integer n, the extension $F(E[n])/F$ generated by the coordinates of the n-torsion points, is finite and Galois. For integers $m \neq n$, we consider when the coincidence $F(E[n]) = F(E[m])$ holds. Daniels and Lozano-Robledo classified coincidence when $F = \mathbb{Q}$ and n and m are prime powers. In this talk, we will describe some preliminary results over a general number field F.