A PRAISE ON PILAR BAYER ISANT

On February 13th 1946 a girl was born in Barcelona and was registered at the Civil Registry as if she were born on February 12th. Her names: Pilar, Teresa, Antònia. Her surnames: Bayer, Isant. Her father was a musician: violin, saxo and clarinet professor from the Liceu and the Municipal Schools of Music in Barcelona, composer and member of the Municipal Band and the Municipal Orchestra of Barcelona. Her grandparents were teachers, as well as an aunt of hers. Therefore, that girl, Pilar, was brought up in an atmosphere where knowledge and culture were quite present; and we have to be grateful to her mother for her willingness to cultivate in Pilar the taste for these values and for her constant support.

Maybe that was a reason why Pilar did her primary studies in a Montessori school, from which she keeps a very good memory. One of the things she always points out as an example of the main characteristics of the Montessori method is the fact that she enjoyed a lot when she learned from her own will; she liked to go to school. And likely she was the apple of her father's eyes, who composed and dedicated a *sardana* to her: *Pilareta*.

Afterwards, she studied high school at the Maragall Institute. There, she met as a teacher Griselda Pascual, very well considered by her family, who knew quite well Griselda's father. At the same time, the young girl Pilar studied music. It seems that when she finished her high school, she had doubts about what to do then: she liked music a lot, and mathematics too. Because, as it is the case for many young people, she wanted to do it all, she wanted to know it all, she didn't want to leave anything behind.

A year after the Vallès floods and the huge snowfall in Barcelona, young Pilar Bayer enrolled at University to study Mathematics; and finished her degree one month after May 1968, one year after being graduated as a piano teacher by The Municipal High Music School of Barcelona. No doubt it could be said Mrs —we would have said Miss at that time— Pilar Bayer became teacher before than graduate.

Herself explains she knew for the first time about Number theory during her last year at university, between the strikes and runaways from that time, in a course in which was planned the study of the French translation of the book on Number theory by Borevich and Shafarevich; but one could only finish the first part, leaving aside the local and analytical methods.

Although many would have considered it to be discouraging or even unimportant, she thought it to be a challenge. Once graduated, in 1968, Pilar became a high school

lecturer; in the morning, at the Juan Boscán Institute, newly founded; and in the evening, at the Maragall Institute, where she had studied as a girl. She even had time, in the afternoons, to teach a mathematics course for students on their last high school year at a private school, and to work as an Assistant professor at the University of Barcelona (UB), firstly in the Pedralbes campus and later at the city centre; she performed her duty as Assistant professor in UB until 1975. In 1969 she was hired also as Assistant professor for the Autonomous University of Barcelona (UAB) until 1977; therefore, she kept working simultaneously for both institutions for long. In 1970, it was the second time that, according to the General Law in Education, were offered some grants for a three-year period of Training for Research Staff. Graduate Pilar Bayer got one of them.

After having finished the text by Borevich and Shafarevich, she and Griselda Pascual, who she had met again at university as Assistant professor, went on studying Number Theory and worked on the book *Corps Locaux*, by Serre, and on the one *Class Field Theory*, by Artin and Tate, for two years. They both also studied other authors, and the result of that work was the writing of their own doctoral thesis. Griselda and Pilar read them on the same day, in April 1975. In that manner, they became the second and the third female doctors in Mathematics at the University of Barcelona (first and second at the Mathematics School), after the remembered Maria Assumpta Català i Poch went, four years before, with an Astronomy thesis (yet at the Sciences School). Pilar Bayer presented the thesis *Extensiones Maximales de un cuerpo global en las que un divisor primo descompone completamente* (Maximal Extensions of a Global Field in which a prime divisor decomposes completely), and Griselda Pascual, *Contribución al estudio de las extensions galoisianas de grupo diedral* (Contribution to the Study of Galois Extensions with Dihedral Group).

After her thesis, doctor Pilar Bayer was one of the first people in our country who did one thing which is very common nowadays, but which was exceptional in those times: to work as a hired lecturer, and do research, for a foreign university. At the German University of Regensburg she became a member of the professor Jürgen Neukirch's team; so, professor Neukirch is mentioned as one of her ascendants in the Mathematics Genealogy Project, the genealogic tree of Mathematics, although officially he didn't appear as director or co-director of her doctoral thesis. Mrs. Pilar Bayer has to be credited with the virtue of knowing how to publicly appreciate people, and be grateful to, who have proved to be generous.

On the occasion of the awarding of the Honor Medal of the *Xarxa Vives d'Universitats* to Dr. Pilar Bayer, which took place in July 2015, and referring to the honoree, Jordi Quer says in his *Laudatio*:

"She belongs to the generation of pioneering scientists who, in a mediocre and sordid environment, began to investigate, to travel to foreign countries to meet the best experts and collaborate with them, to publish results in specialized journals and to explain them in international conferences. Thanks to people like her, university and research in our country made a leap forward during the 80s and the 90s, and also at the beginning of this century, placing Catalonia in the world map of scientific research."

The first Dr. Pilar Bayer's publications in very prestigious and internationally renowned journals are about special values of Zeta functions, about special values of L-functions and Iwasawa theory, and also about automorphic forms and Hodge theory, and all of them dating from that time. Since the beginning she understood, and also did know how to communicate, that the algebraic methods can't be separated from the study of functions because many properties of the algebraic numbers are very well codified in functions with good analytical properties. I would also dare to say, whether we can talk about a unifying thread between her research and her desires to increase her mathematical knowledge, that this is precisely the interaction between algebra and the study of functions with arithmetical meaning.

I would not like to move on without giving an example of the potential of her research and its original results. In her first article written jointly with Jürgen Neukirch, they both constructed a theory with which it was proven that a Lichtenbaum's conjecture relative to some special values of the Zeta function of a number field is a consequence of the main conjecture of Iwasawa theory. The result was completed later on by Mazur and Wiles in one of the more celebrated articles of the XXth century's last period: the proof these two authors publicated in 1984 of the main conjecture of Iwasawa theory for Abelian extensions of the field of rational numbers. Yet, thirty-six years later, and after it was applied by authors such as Soulé (even before Mazur and Wiles), Schneider, Milne, Étesse, Neukirch or Fontaine, it has been also used by the female mathematician Rin Sugiyama in the proof of a Tate conjecture for products of Fermat varieties over finite fields.

Before her fourth year living in Germany, she obtained an official post as an Associate professor of Algebra at the University of Santander, by previously sitting an examination. And at her arrival, she was nominated Director of the Department. One year later she moved to the Autonomous University of Barcelona, and the next year, in 1982, she became Professor of Algebra at the University of Barcelona, job she still has at present.

I had seen professor Pilar Bayer only for one hour on the second day of my first year in the Mathematics School —the first day for me— at the UB, in October 1974. At that time she was the professor in charge of the course. Nevertheless, the next day I changed group and I didn't see her any more until December 1982. I had already finished my degree and was teaching as a training assistant professor at the *Departament d'Àlgebra i Fonaments*, and Dr. Pilar Bayer went there as a Professor, and with a three-month baby, circumstance that obliged her to leave on time the doctorate course she had recently begun: "*Global Class Field Theory*". This is another trait of hers I would like to pay

attention to: her willingness to give herself to others. At the School there weren't many doctorate courses, and Pilar Bayer considered she had to make a difference from the very first moment.

From that year on, never failed in the School any doctorate course on Number theory; and at first, all given, or at least organized, by her. All courses were about different subjects; for instance, in 1983-84, the following year, Dr. Pilar Bayer gave a doctorate course, formally titled *"Automorphic Forms and adele groups"*, in which she explained the theorems recently proven by Faltings about the Mordell conjecture and which would entitle him to the Fields Medal in 1986. The doctorate courses on Number theory were tremendously successful amongst students because the programme was changed every year. Dr. Pilar Bayer honored in this way the academic liberty and decided which contents would be explained every year and the programmes to be followed.

And then, the first doctoral thesis. In her first period at the Autonomous University of Barcelona, she had had Núria Vila and Enric Nart, amongst others, as last year students. At the UAB, there was also another professor specialized in Number theory, Dr. Pascual Llorente. Dr. Pilar Bayer directed the doctoral thesis of Núria Vila, and Dr. Llorente that of Enric Nart. Some publications written in collaboration are from that time. Núria Vila wrote her thesis on the inverse problem of Galois Theory and presented it in June 1983 at the UAB.

Àngela Arenas's doctoral thesis came from an arithmetic algebraic problem, whose solution showed of more analytical bias, about the exact values of a parameter to which some results of Núria Vila's thesis can be applied. When doing her doctoral thesis, Àngela Arenas made such a display of happiness that often culminated in laughing fits, which were heard by those who had the office next to Pilar's. Àngela Arenas read her thesis in May 1985 and became officially the second Dr. Pilar Bayer's PhD student.

As a result of Núria Vila's thesis, contacts about other subjects and projects were established with professor Jean-Pierre Serre, and those contacts were to be maintained for the years to come. It is precisely about a deep study on some immersion problems, on the context of the inverse problem of Galois theory, that Teresa Crespo's thesis was done, next directed by Dr. Pilar Bayer.

My acquaintances know I am a stubborn person and that it is hard to make me change my mind or to make me see that I'm wrong. I also had the pleasure to have Dr. Pilar Bayer as the director of my thesis. And she did it while she was also directing Teresa Crespo's thesis. Some days after asking her to be my director, I presented her the problem I intended to work on and which, naturally enough, was not the problem she had the intention to give it to me. After let me talk, she presented her problem to me, but I insisted in mine. She, with great generosity, directed my thesis in the problem I wanted. Teresa Crespo and I defended our thesis in February 1988, on the same day, such as she and Griselda Pascual had done nearly thirteen years before. Teresa Crespo's thesis was awarded with the Doctorate Extraordinary Award of the Mathematics School.

Pilar Bayer willingness to work for others is another prayseworthy aspect of her career. During the time she lived in Germany she worked very hard and collaborate with very renowned mathematicians. And without thinking only on her curriculum vitae nor fighting for making a preeminent place for herself in the international mathematical community, she decided to serve others, and no doubt that in doing so, she also gained for herself a remarkable position. She was told one day, more or less: "you should put some order in that department and try to get some more research be done in it". And she felt she could serve the University of our country. She launched a good effort without having to leave any of her close and productive collaborations. I would also like to thank her publicly for her generosity. Professor Pilar Bayer has devoted herself to university in its literal sense.

After the doctorate courses on Number theory in 1985-86 were taught by doctors Llorente and Nart, time had arrived to make a decisive step. In our country every year there were more people interested in Number theory (and it is easy to guess why, isn't it?). It was the time of the constitution of the *Seminari de Teoria de Nombres*. The first formal edition of the seminar took place in 1986-87 and the subject was "Rational Points of Algebraic Curves"; Frey and Serre's strategy that in 1995 should lead to Wiles's proof of Fermat Last Theorem was presented. At the same time, Dr. Pilar Bayer was not only responsible for the programme and the direction of the seminar, but she also lectured a doctorate course; and her students' presentations were added at the end of the seminar as if they were a part of a workshop on "Artin Representations and Elliptic Curves". This turned out to be fundamental in the future because some subjects of doctorate courses would be included in the *Seminari de Teoria de Nombres*, and would help to keep it going on for the first years, due to the increase of students attending to her lectures.

At that time preliminars were settled that lead to Joan Carles Lario, Josep González-Rovira and Anna Rio's PhD thesis. Joan Carles Lario defented his thesis in September 1991, and Josep González-Rovira presented his in December 1993 defending it a few weeks later. It was the same for Anna Rio, who presented her in December 1995 and defended it some weeks later. Joan Carles Lario's thesis, about Serre's Modularity Conjecture, was honored with the *Josep Teixidor Award* of the *Societat Catalana de Matemàtiques*.

After that first formal edition of the *Seminari de Teoria de Nombres*, its work do not stopped. Subjects such as "Modular Curves and Eisenstein's Ideal", in 1988-89, or "Hodge-Tate's Galoisian Structures", in 1989-90, were the prelude for Dr. Pilar Bayer, in 1990-91, "put all of us to work on the machine". She indeed invented a subject,

Corbes modulars (algoritmes i taules), and its objective was that every person at the seminar, from the youngest to those not so young and whether they had had or not any experience using computers, could devote ourselves to make explicit calculations with computers. And this was the melting pot where the first publication of the *Seminari* took place: a book that contains calculations made by more than one score of twenty people over different aspects of modular curves. This book, which was published in 1992, would be a starting point for future research and would be useful for the thesis directed by Dr. Bayer herself or by other doctors from the seminar.

Soon the echo of the *Seminari* crossed borders, and in 1989 those European circles working on Number theory talked about having Barcelona as the seat of the *Journées Arithmétiques 1991*. This biennial conference is organized alternatively in a French city and in another European city out of France. At the *Journées* of Bordeaux 1993 professor Pilar Bayer was elected to be the organizer, in 1995, in Barcelona, of the *XIX Journées Arithmétiques*.

Meanwhile, the 1991-92 seminar "Arithmetical Surfaces" was the starting point for another doctoral thesis directed by Dr. Pilar Bayer: Jordi Guàrdia's thesis, defended by him in January 1998. Dr. Bayer would combine direction of this thesis with the organization of the *Journées Arithmétiques* of Barcelona and its subsequent tasks of preparing and editing the proceedings.

The course 1992-93 was a turning point for the *Seminari de Teoria de Nombres*. Many people participated and their interests were very varied. It was decided the programme would cover two subjects instead of one alone, as it had been usual until then. Dr. Pilar Bayer was in charge of the programme and the preparation of the subject "Galois Representations of dimension 2", whereas Dr. Enric Nart was in charge of "Chow Motives".

The course 1993-94, with the initial work of preparation of the *Journées Arithmétiques 1995*, did not have as many sessions as usual, but they were very intense, as the subject clearly shows: "Preliminaries to the Fermat-Wiles theorem". Someone said, when the course finished, that amongst the 300 people in the world capable of completely reading Wiles' proof of the Fermat Last theorem, 30 of them were located in Barcelona. Probably this someone was right in the 30 (whereas 300 perhaps was a figure a little pessimistic).

During the course 1994-95, although the very important task of organizing the *Journées*, that occupied more than half the sessions, the seminar went on and Dr. Pilar Bayer programmed the different subjects about "Arithmetic of Number Fields" which that course were studied and worked on.

The course 1995-96 was signed by renewal. Dr. Pilar Bayer along with Dr. Teresa Crespo had to edit the proceedings of the *Journées Arithmétiques* and let Jordi Quer, Joan Carles Lario and Anna Rio the organization of the seminar of that course, which consisted on checking in detail the proof of the modularity of some specific elliptic curves so it was like the course of the problems of the seminar over the theorem of Fermat-Wiles. That was also the first seminar whose sessions wasn't completely celebrated in the University of Barcelona premises.

The Fermat theorem had been proved already and we had studied its proof. And Dr. Pilar Bayer considered it was time for all of us to prepare for a new challenge. Therefore she dedicated the seminar 1996-97 to the subject "Automorphic Representations of GL(2)", which had to open new paths to fully understand some very important arithmetic conjectures that, yet still, remain open.

The next two courses, and while she directed Montserrat Alsina's thesis, Dr. Pilar Bayer took a step aside and let the organization and coordination of the seminar's sessions to others. Consequently, during the courses 1998-99 and 1999-2000 the seminar was an intensive week in the style of conferences, and its sessions were celebrated at the *Escola d'Alta Direcció i Administració* (EADA), in Collbató.

Henceforth every seminar has had this specific characteristic of being organized as a Conference concentrated in a week, with at least one subject, and being programmed by one or more people. Amongst the twenty-seven subjects programmed during the fifteen seminars from 2000-01 until now, not mentioning the isolated communications, Dr. Pilar Bayer has participated directly in the programming of, at least, six of them. And she has given in them a minimum of sixteen plenary lectures. Some of these seminars/conferences have been international; in fact, we could say that from 1999-2000 on, almost all of them have had some lecture given by a foreign researcher, or have been clearly international.

Montserrat Alsina's thesis was presented in December 1999 and defended a few weeks later. This thesis, which obtained the *Josep Teixidor Award* from the *Societat Catalana de Matemàtiques*, was the first Dr. Pilar Bayer directed on the subject of Shimura curves; it would also be the subject for Victor Rotger's thesis, presented in November 2002 and defended in January 2003, and that was awarded with the Doctorate Extraordinary Award of the Mathematics School and also with a secondary award at the *IX Premi del Claustre de Doctors de la UB* to the best doctoral thesis in 2005. The main results of Montserrat Alsina's thesis were published in 2004 by the American Mathematical Society in the monograph *Quaternion orders, quadratic forms and Shimura curves*, by Dr. Montserrat Alsina and Dr. Pilar Bayer.

These last years Dr. Pilar Bayer has yet directed Iván Blanco's thesis, defended at the Universidad Complutense de Madrid in July 2012; the one of Paloma Bengoechea, co-

directed with professor Don B. Zagier and defended in Paris in 2013, and last but not least, Dionís Remon's thesis, defended fifteen days ago, and Piermarco Milione's one, to be defended tomorrow, on January 29th. At these moments, Dr. Pilar Bayer is also co-directing Laia Amorós's thesis with professor Gabor Wiese, from Luxemburg University.

Without Dr. Pilar Bayer nothing of that would have been possible. Maybe she would say that she never ever could have achieved something like that without the rest of the seminar collaborators taken as a whole, and maybe she would be right. But one thing is for sure: all of that wouldn't have been possible without her. Her guidance, her ability to detect which are the interesting problems and also to know who is able to work on them with the best probability of success —I would say with an absolute certainty of success—, her signs of courage in difficult moments, and her unconditional dedication to the whole task make her irreplaceable. And, in her case, the expression "no one is irreplaceable" would not work; it would be literally false.

Following the thread of this story one could be led to a misunderstanding and think that Dr. Pilar Bayer has been only devoted to direct doctoral thesis and the Seminari de *Teoria de Nombres.* Nothing further from the truth. Leaving aside the deepness of her original research results, of which I have done a little mention above, and which are undoubtedly a legitimate source of pride for Dr. Pilar Bayer and those who share seminar with her, she has participated in a minimum of seventeen funded research projects, each of them of a different subject, of which she has directed a minimum of twelve. She has participated in many examination boards to take new faculty professors; in fact, one could say, if I'm allowed to exaggerate a bit, that for many years there weren't many examination boards taking Algebra professors which Dr. Pilar Bayer was excluded of. The same could be applied for many faculty seats. And not only within the Spanish borders; she has also been a member of committees assigning professor and lecturer seats abroad, particularly in United Kingdom. For instance, the Savilian Chair of Geometry or the Professorship of Pure Mathematics, both of them in the University of Oxford, or the Lucasian Chair in the University of Cambridge. On the other hand, she was hired in 2004 as an Emmy Noether Professorin in the Georg-August Universität Götingen, in Germany, during the unique sabbatical period she has enjoyed at our University, and which lasted only a semester.

Pilar Bayer has been a member of the editorial board of international Mathematics journals and she still works actively for three of them. She has been invited as a lecturer many times, or as a speaker, for many international scientific conferences which I'm not going to get into detail. Science in general, Mathematics in particular, and above all, Number theory promoter, Pilar Bayer has so many merits I will not try to go into detail. I will only say she has participated in many round tables, some of them international, she's also being requested by newspapers and radio and television broadcasts to give her opinion over our few achievements as mathematicians that from time to time appear in the media, and she is often invited to give speeches and publicize science at all levels, from a university and academic register to a more popular one. She is also well known and awarded for this task. For instance, in April 2015 she received the Women Medal 2015 of Sarrià-Sant Gervasi District. As a UB press review reflects, concerning the event:

"the award gives reconnaissance to Dr. Pilar Bayer's career and compromise to divulge the women role in the history of Mathematics, Science and Humanistic knowledge."

Her dedication to the university management has not been idle. From director of Department, on more than one occasion, or member of the Consultative Committee of the UB University Management Council from 2004, she has also made a difference in the Academic Committee or the Complaints Commission, all of them belonging to the UB, and her presence in the management bodies of the University has been constant. In fact, she was the Director of the *Departament d'Àlgebra i Fonaments* when in 1986 and 1987 there was a restructuration of the Mathematics School Departments and that department and the one of *Geometria i Topologia* were merged into one: the current *Departament d'Àlgebra i Geometria*.

All these examples are only a sample of the enormous working ability of Dr. Pilar Bayer of which I think praiseworthy and necessary to remark. To make it more evident, I could add even that while she was preparing the *Jornades Aritmètiques*, directing one doctorate thesis, programming the *Seminari de Teoria de Nombres* and the corresponding doctorate courses, and carrying up the rest of the academic, management and research tasks of the University, in 1994 she was nominated Academician of the *Reial Acadèmia de Doctors*, where she entered officially as fully-fledged member giving a speech on the Platonic Solids in January 1996. During the event, in his responding speech to the Hon. Dr. Pilar Bayer i Isant, the Hon. Dr. Ricard Garcia i Vallès said:

"The *laudatio* of the newly-elected member does not constitute any problem in view of Dr. Pilar Bayer i Isant's scientific value. But it is a serious problem when I come to its exposition, by hers quantity and quality."

And last week it's been twenty years since those words!

This is not the only honor or award Dr. Pilar Bayer has received. In 1997 she was appointed elected academician by the *Reial Acadèmia de Ciències i Arts*, where she was officially received in February 2001 and gave a beautiful and deep speech about "Poincaré and the Fuchsian Functions".

The Generalitat of Catalonia, in his decree 144/1998, June 9th 1998, awarded the Narcís Monturiol Medal to scientific and technological merit, amongst others, to Pilar Bayer Isant, "Algebra Professor of the University of Barcelona. For her contribution to scientific advances of international impact, for her teaching career, and for her dedication to train research staff in the field of Arithmetics".

In February 2010, one day in which in Madrid was raining heavily, Dr. Pilar Bayer read her entrance speech as a full member of the *Real Academia de Ciencias Exactas, Físicas y Naturales*, intitution of which she was an academician from 1994. The impressive speech she gave was about "Antecedentes y evolución de la teoría de la multiplicación compleja".

I would also like to mention that from June 2001 on, the most illustrious Mrs. Pilar Bayer is included as a fully-pledged member of the *Institut d'Estudis Catalans*, into the Science and Technology section. Her collaboration with this institution dates from long time ago, because of her association to the *Societat Catalana de Matemàtiques* and also for the many articles she has published in its *Butlletí* and in the journal *Notícies*, both of them belonging to this society, which is subsidiary to the Institute.

Dra. Pilar Bayer's scientific career does not end in her membership and the task she does in these above-mentioned academies. In the introduction of her version of *Disquisicions Aritmètiques*, Griselda Pascual wrote:

"The idea of creating a Catalan version of the *Disquisitiones* came, some years ago, from a conversation with Dr. Pilar Bayer. We thought that having this text in Catalan would be an invaluable help for those who, attracted to Mathematics, wanted to enjoy, first-handedly, Gauss genius."

Therefore, Griselda Pascual worked, around 1989, on a translation into Catalan of the *Disquisitiones Arithmeticae* by Gauss based on different translations; essentially, one in English and another one in French, although there was also another one in German. Dr. Pascual and Dr. Bayer got into terms to try to publish the translation. The project counted with the collaboration of mathematicians and linguists to check the text scientifically and linguistically. They devoted six years to adapt the translation to the original Latin of the work. It is obvious to mention that Pilar Bayer's direction was fundamental for the success of the project. Finally, the *Disquisicions aritmètiques*, of 682 pages, was published by the *Institut d'Estudis Catalans* in 1996.

Another Dr. Pilar Bayer's work of historical nature came to light due to two research projects of the *Institut d'Estudis Catalans* she directed. As one can read in her introduction, the work is "...an approach to a selection of mathematical works that, being developed in Germany between 1850 and 1950, have had a great influence in some of the great successes of contemporary mathematics." We are referring to the work *Arrels germàniques de la matemàtica contemporània. Amb una antologia de textos matemàtics de 1850 a 1950.* ("German Roots of Contemporary Mathematics. With collected mathematical texts from 1850 to 1950") Published by the Institute in 2012, it reflects in its 782 pages nearly six years of work: translation, documentation, study and creation of argumentative, mathematical and historical, texts about some of the great works and personalities of the Mathematics developed in Germany for a century. The

work was acknowledged with the *Premi Crítica Serra d'Or 2013* in the section Research and Other Sciences.

I would like to finish this praise with some words about the huge task developped by Professor Pilar Bayer in the field of university teaching. If all that has been explained from her until now had taken place in research centres or without any teaching task, in such a way that the balance between the time devoted to research and the one devoted to teaching were inclined favouring research activities, her work would have to be considered really huge. Nevertheless, as it has been usual in our country up to the present moment, she could only devote herself to research combining it with teaching and management tasks: mathematical research in our country is done at university, "inbetween classes", and it has been necessary to be competitive in these circumstances.

Luckily for her students, Dr. Pilar Bayer's classes and courses have been, and still are, masterful in its literal sense. At every moment, she has also known how to innovate the degree subjects as much as possible without disturbing the system, and has achieved a spectacular success. After the renewal of some between the more traditional courses by means of varying the programme in-depth, and while it wasn't still consolidated, she invented some optional credits with a content that should be known for most of the graduates in Mathematics; she even proposed some optional courses to be studied during the first cycle, against the usual practice of the time, which restricted the options to the second cycle.

She also elaborated optional credits for a wide range of public which destroyed the nearly paradigmatic believe that Number theory was only a theoretical subject with nothing to do with any practical application. In this way, she succeeded in having the highest number of undergraduates registered in their optional credits. And when the degree underwent a radical change, upon the creation of post-graduate courses and masters, she knew how to reformulate many of the contents belonging to the optional credits and include them in the compulsory or core subjects of the degree.

One of the most beneficial tasks in this sense, due to the amount of work she did, is the direction of academic works in the second cycle, at the end of degrees, or at the end of masters. Apart from directing works on Number theory, she has directed other subjects such as Coding Theory and Cryptography, Mathematical Economy, Lie groups and Algebras, Theoretical Physics, Mathematics applied to Music, Differential Operators and, last but not least, Computer Science.

Her students are very grateful for her mastery. I remember well one student of her Number theory class that, just when the last class of the course finished, literally told me:

"It's been a great luxury to have had Dr. Bayer as a professor..."

And I would add: "It is a fantastic luxury to count on Dr. Pilar Bayer as a professor and as a friend."

You should have noticed that all along this praise I have used deliberately different ways to treat and honor Pilar Bayer: girl, young girl, young, Mrs. (and Miss), person, professor, graduate, doctor, researcher, Illustrious Mrs., Hon. Mrs., magister and friend, but there is one word I have left for the end.

Dearest Pilar, thank you very much for all your heritage.

Artur Travesa Barcelona, November 2015 and January 28th 2016. Corrected version: April 2016